Analysis and Verification of an Optimal Design Solution for Rubble Mound Breakwaters Considering Interactions of Failure Modes

Yunce Zhang[1]; Zongmin Liu[2]; Yafei Wang[3]; and Changguo Wang[4]

Abstract: As an essential coastal structure, a breakwater is generally designed to be stable and reliable during its lifetime. With respect to stability, failure modes usually are statistically independent. However, the interaction of different failure modes can produce extra effects for the calculation of the total cost. Considering interactions of failure modes, this paper presents an analysis and verification of an optimal design solution for rubble mound breakwaters to minimize the total cost, including construction and repair costs. The three main failure modes, overtopping, armor instability, and crown wall sliding, are involved in the solution. An auxiliary coefficient representing the extent to which the failure mode (or interaction) affects the structure was used to calculate the extra repair cost along with the failure probability in the cost optimization. The aforementioned solution was applied to a real breakwater example, and a sensitivity analysis of the total cost of the design variables was carried out. The total expected cost using the optimal design solution was 19.1% less than the cost before optimization. Other coastal structures under wave impact loads can have the same failure modes and also could be optimized by the solution. The solution in this study can provide economical design recommendations. **DOI:** 10.1061/(ASCE)WW.1943-5460.0000544.

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Introduction

Breakwaters are major marine structures in any harbor, and their function is to resist waves. Over time, breakwaters disintegrate, with many adverse effects. On March 11, 2011, due to damage to the breakwater in the Tohoku region, many great losses, including nuclear leakage, occurred (Jayaratne et al. 2016). To avoid structural damage, it is important for researchers to consider the failure modes in the design of breakwaters.

There are many studies of failure modes of caisson-type breakwaters under wave impact loads (Chaudhary et al. 2017; Elsafti and Oumeraci 2017; Esteban et al. 2018; Mitsui et al. 2016; Wang et al. 2016). Guanche et al. (2013) proposed an effective method considering waves imposed on vertical structures, and illustrated that the method can reconstruct time series of stability parameters on a vertical breakwater. Rubble mound breakwaters, a common type of breakwater, are widely used worldwide. The stability and reliability of a rubble mound breakwater are connected to its resistance to failures imposed by waves. However, there are limited studies into the failure modes of rubble mound breakwaters. There are various failure modes corresponding to the several components of a rubble mound breakwater, such as geotechnical instabilities, movement of the armor, sliding of the crown wall, and erosion of the toe or berm (Ergin and Balas 1997). For simplification, failure modes of a rubble mound breakwater are mainly classified into three types: overtopping failure, armor failure, and crown wall sliding failure (Castillo et al. 2004). Minguez et al. (2013) considered only two failure modes for breakwaters, which in fact is too simplistic to explore the failure mechanism of breakwaters fully. Lee et al. (2018) conducted a poor analysis with numerical results with only general conclusions. Somervell et al. (2017) did not consider the failure mechanisms of breakwaters. Hoby et al. (2015), did not methodically analyze the failure mechanisms.

Minguez et al. (2013) proposed an available and efficient algorithm to solve a design problem using Benders' decomposition. They considered the design problem as a bilevel problem and addressed it by means of establishing an objective function and a group of design and actual constraints. Failure probabilities were presented and evaluated through the first-order reliability method (FORM). They assumed that the optimal design and the analyses of reliability are decoupled, which can make computation and the algorithm more efficient. They noted that the engineering problem of optimal design can be seen as an optimization problem which is constrained through different associated optimization problems.

Hoby et al. (2015) provided the optimal design of breakwaters through a probabilistic analysis. In this optimization problem, minimum cost was calculated using interrelated geometry constraints and safety margins. Based on several failure modes, the procedure was applied to a case study under construction at Thalai Fishing Harbor. Hoby et al. provided three modes of failure, sliding failure, overturning failure, and failure of the foundations regarding stability defined by Guanche et al. (2013). To alleviate the need for arbitrary selection of the slope, the sea-side slope of the breakwater

[1]Postgraduate Student, College of Aerospace and Civil Engineering, Harbin Engineering Univ., Harbin 150001, People's Republic of China. Email: zhangyunce@hrbeu.edu.cn

[2]Associate Professor, College of Aerospace and Civil Engineering, Harbin Engineering Univ., Harbin 150001, People's Republic of China. Email: liuzongmin@hrbeu.edu.cn

[3]Postgraduate Student, Centre for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150080, People's Republic of China. Email: 14S018027@hit.edu.cn

[4]Professor, Centre for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150080, People's Republic of China (corresponding author). Email: wangcg@hit.edu.cn

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was included in the established procedure for the calculation of cost optimization. Therefore, their method does not use the FORM and had an assessment for the design.

A reliability analysis for two different caisson-type breakwaters was carried out by Lee et al. (2018). Considering design wave force, they defined the limit state function and investigated the change of the failure probability due to the reduction of wave force. The reliability index of two different types of breakwater was compared. Lee et al. concluded that the reliability of a caisson-type breakwater is affected by several factors, including the mean peak increase factor, the mean wave direction, and the mean of maximum spreading parameter.

Based on a multiobjective optimization procedure, Somervell et al. (2017) introduced a methodology to design a vertical breakwater. Through a theoretical model, two objective functions, i.e., wave energy dissipation coefficient and material volume in the vertical breakwater, were calculated. The theoretical model was validated by comparison with previous papers, and the multiobjective optimization tool using a genetic algorithm (GA) was applied to satisfy two conflicting objectives as much as possible. However, the study did not take into consideration the failure mechanism of breakwaters, and the practical constraints were poorly described. Despite extensive studies of failure modes and failure probabilities, such as at Thalai Fishing Harbor (e.g., Hoby et al. 2015) and in the Tohoku region in Japan (e.g., Jayaratne et al. 2016), and despite recent advances in optimized modelling (e.g., Minguez et al. 2013; Somervell et al. 2017), the failure probabilities of breakwaters still have not been fully elucidated by researchers, and some essential issues relating to interactions among modes of failure included in the optimization process under wave action remain unsolved.

This study introduces a new parameter to consolidate and describe the interactions among different modes of failure in a rubble mound breakwater. Furthermore, a complete and comprehensive framework was constructed to interpret failures of a rubble mound breakwater in an optimization model. Moreover, according to the probabilistic approach, the proposed framework is demonstrated through the optimal engineering design of a practical breakwater example. The results of this research can promote the systematic development of optimal design against failures of a rubble mound breakwater and provide design recommendations to minimize the total cost of a breakwater from the perspective of stability and reliability.

Methodology

Optimal Probabilistic Method

The problem structure is as follows:

$$\underset{x}{\text{minimize}} f_1(x,z) + \sum_{j=1}^{n} f_2(x,y,z) \tag{1}$$

subject to

$$g(x,y,z) = 0 \tag{2}$$

$$k(x,y,z) \leq 0 \tag{3}$$

where $f_1(x,z)$ = construction costs; $f_2(x,y,z)$ = repair costs for failure modes; $g(x,y,z)$ and $k(x,y,z)$ = equality and inequality constraints; and x, y, and z = design variables, random variables, and auxiliary parameters, respectively. In the repair costs, the process of the probabilistic method of structural reliability analysis related to the rate of failure in one mode of failure is illustrated (Burcharth 1997). In the construction costs, the change of structural sizes occurs with variation of constraints. Each failure mode is determined by a corresponding limit-state equation (Ditlevsen and Madsen 1996). Under failure mode m, a group of design variables is given and the probability of failure P_f is expressed as (Castillo et al. 2006)

$$P_f = \text{Prob}[g_m(x_1, x_2, \ldots, x_n) \leq 0] \tag{4}$$

where $(x_1, x_2, \ldots, x_n)$ = values of variables included and

$$g_m(x_1, x_2, \ldots, x_n) = h_{sm}(x_1, x_2, \ldots, x_n) - h_{fm}(x_1, x_2, \ldots, x_n);\ m \in M \tag{5}$$

where $g_m(x_1, x_2, \ldots, x_n)$ = safety margin; $h_{sm}(x_1, x_2, \ldots, x_n)$ and $h_{fm}(x_1, x_2, \ldots, x_n)$ = two opposing magnitudes of generalized forces that incline to prevent and cause the related failure mode, respectively; and M = group of all modes of failure.

It is assumed that failure happens during storms and storms are supposed to be random processes. If the essential variables of waves satisfy $g_m \leq 0$, structural failure will occur. Then, under failure mode m, the probability P_{fm} is written

$$P_{fm} = \int_{g_m(x_1, x_2, \ldots, x_n)} f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \tag{6}$$

where $f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n)$ = joint probability density function for all variables included in the problem.

To calculate the failure probabilities, the nonlinear constrained optimization problem is solved according to the reliability analysis method for structures. The reliability index is as follows:

$$\begin{aligned} &\min \beta_m = \sqrt{\sum_{j=1}^{n} y_m^2} \\ &\text{s.t.} g_{Y1}(y_1, y_2, \ldots, y_n) \leq 0, \\ &g_{Y2}(y_1, y_2, \ldots, y_n) \leq 0, \ldots, g_{Ym}(y_1, y_2, \ldots, y_n) \leq 0 \end{aligned} \tag{7}$$

where β_m = reliability index for failure mode m. Hence, through the theoretical derivation, the failure probability can be expressed equivalently as follows:

$$P_{fm} = 1 - \Phi(\beta_m) \tag{8}$$

where $\Phi(\cdot)$ = standard normal cumulative distribution function. Because the calculation of the failure probability is difficult to solve using the definition of the failure probability, the calculation of the failure probability can be equivalent to the solution of the multidimensional normal distribution function.

The specific solutions of the reliability index and the failure probability are elaborated in the optimal design solution and the Appendix. As random variables, the wave parameters such as H, T, and η, follow a normal distribution, and the probability density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)}{2\sigma^2}\right], -\infty < x < +\infty \tag{9}$$

where μ = mean of a random variable; σ = standard deviation of a random variable; and x is used as a random variable of the distribution.

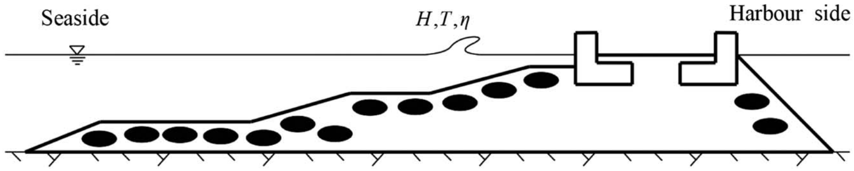

Fig. 1. Cross section of a rubble mound breakwater.

Failure Modes

There are various failure modes to consider, for example, geotechnical instabilities, movement of the armor, sliding of the crown wall, and erosion of the toe or berm, corresponding to several components of a rubble mound breakwater (Ergin and Balas 1997). To simplify the calculations, three failure modes of a rubble mound breakwater are classified in the optimal design solution: overtopping failure, armor stability failure, and crown wall sliding failure (Castillo et al. 2004). A cross section of a rubble mound breakwater is shown in Fig. 1.

Each mode of failure is usually determined by a discriminant function or a threshold criterion, which provides a quantitative indicator of whether the failure mode has occurred. However, a rubble mound breakwater made up of stones of different sizes is a granular system which is maintained by structural stress transmission and geometrical connectivity because of friction and interlocking among stone units. In marine structures, structural failures do not necessarily result in complete destruction but in irreversible structural damage, which leads to a sharp decline in their resistance and performance.

Because the overall structure includes several components, failure in one component can easily cause failure in another component. Some modes of failure are correlated with one another. Due to complex factors and a lack of data from a sufficient number of engineering examples, the interaction among modes of failure is extremely complex and difficult to study, and some essential issues related to the interactions among modes of failure under wave action remain unsolved. In this study, the interaction among modes of failure is only considered for simplicity. Furthermore, a complete and comprehensive framework is illustrated to interpret failures of a rubble mound breakwater in the optimization model.

Overtopping Failure

Some essential parameters exist in the method, such as the slope of a breakwater $\tan\alpha$, freeboard F_c, wave height H, and wave period T. The waves rise beyond the assumed extension of the mound slope and the freeboard of a breakwater is considered to be an indirect sign which indicates an overtopping condition. Thus overtopping failure (Failure O) occurs when the value of wave run-up R_u is greater than the value of freeboard F_c, where wave run-up R_u is the maximum excursion of water over the assumed slope (Fig. 2).

According to experimental results, Losada and Herbich (1990) provided the following discriminant function to calculate the dimensionless quantity R_u/H:

$$\frac{R_u}{H} = A_u(1 - e^{B_u I_r}) \tag{10}$$

where A_u and B_u = coefficients subject to armor units; and I_r = Iribarren number.

Hence, the limit-state equation is as follows:

$$k_o = A_u(1 - e^{B_u I_r}) - \frac{R_u}{H} \tag{11}$$

Failure due to Armor Instability

Armor instability failure (Failure A) means that the stones placed on the armor have large displacements from the original position and even become separated from the armor (Fig. 3). According to experimental results, Losada and Herbich (1990) provided the following discriminant function to calculate the dimensionless quantity:

$$\frac{W}{\gamma_w H^3} = R\Phi_e \tag{12}$$

where Φ_e = stability function; γ_w = water unit weight; R = dimensionless constant considering γ_c and γ_w; and W = individual armor block weight.

Hence, the limit-state equation is as follows:

$$k_a = \gamma_w R\Phi_e H^3 - W \tag{13}$$

Failure due to Crown Wall Sliding

Crown wall sliding failure (Failure C) occurs when there is a sliding displacement at the bottom of the crown wall owing to the action of the water pressure force [Fig. 4(a)]. Crown wall sliding failure can be expressed by the following verification equation (Castillo et al. 2004):

$$\mu_c(W_1 - F_v) = F_h \tag{14}$$

where μ_c = friction coefficient (Martin et al. 1999); F_h = horizontal force because of water pressure; F_v = total vertical force because of the water pressure; and W_1 = actual crown wall weight, including

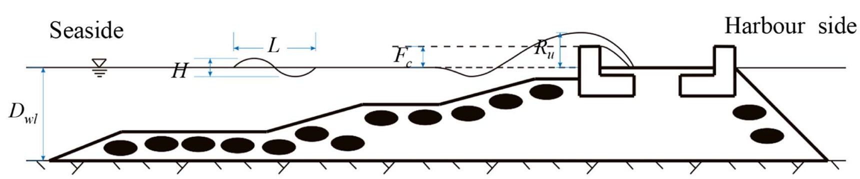

Fig. 2. Overtopping failure.

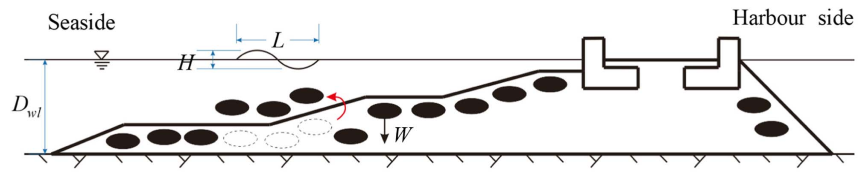

Fig. 3. Armor instability failure.

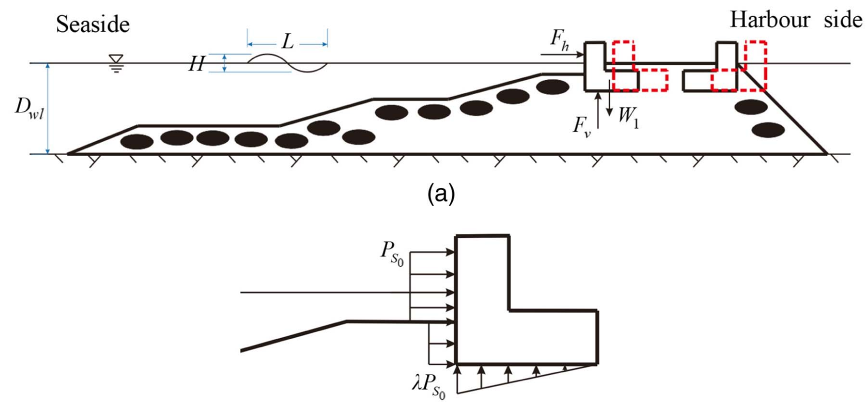

Fig. 4. Crown wall sliding failure: (a) failure due to crown wall sliding; and (b) crown wall.

dry and submerged parts. The detailed formulas [Fig. 4(b)] are given in the Appendix.

Hence, the limit-state equation is as follows:

$$k_c = F_h - \mu_c(W_1 - F_v) \tag{15}$$

Example of Rubble Mound Breakwater

Based on the reliability analysis, the optimal design solution is proposed to be applied to a rubble mound breakwater (Castillo et al. 2004, 2006; Minguez and Castillo 2009; Minguez et al. 2013). The purpose of optimal design is to realize a balance between performance and cost, which ensures enough performance to resist waves and minimizes the total cost of a breakwater as much as possible. For a comprehensive consideration, overtopping failure, armor instability failure, crown wall sliding failure, and the interactions among these three modes of failure are included. For the sake of generality, the design variables consist of major parameters and structural parameters, such as the freeboard F_c and the mound slope angle $\tan\alpha$. A detailed description of the parameters is given in the practical breakwater example.

The solution of optimal design is applied to the example of a rubble mound breakwater. The variables and parameters are provided in Fig. 5.

Description of Variables

Design Variables for Optimization

The necessary information for the design variables used in the rubble mound breakwater is given in Table 1. The design values in the actual engineering are used as the initial values. Taking into consideration the practical engineering application, the values of lower boundary and upper boundary are provided so that reasonable ranges are not exceeded. These design variables affecting the stability and reliability of a breakwater can be optimized to reduce the total cost as much as possible.

Random Variables

The mean and the standard deviation of the random variables are presented in Table 2. All three random variables follow a normal distribution. Considering offshore wave statistics, the mean of the wave height and wave period is 3 and 8, respectively. The storm surge illustrates the change of the water level considering atmospheric situations, which is selected by assumed mean and standard deviation.

For wave random variables, the equations are as follows:

$$\left(\frac{2\pi}{T}\right)^2 = g\frac{2\pi}{L}\tanh\left(\frac{2\pi(D_{wl}+\eta)}{L}\right) \tag{16}$$

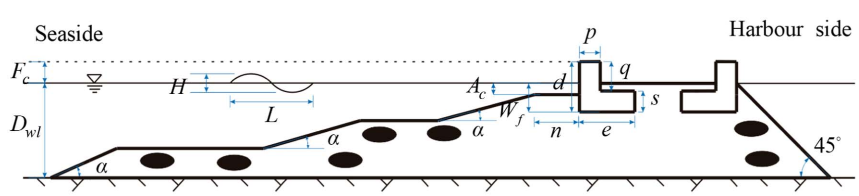

Fig. 5. Rubble mound breakwater with parameter definitions used in the example.

Table 1. Values and meanings of design variables

Design variable	Initial value	Lower boundary	Upper boundary
Freeboard, F_c (m)	3	2	5
Rubble mound slope angle, $\tan\alpha$	0.3	0.2	0.5
Height of crown wall, d (m)	5	4	6
Berm level, A_c (m)	1	0	2
Berm width, n (m)	3	1	5
Crown wall width, e (m)	4	3	5

Table 2. Values and meanings of random variables

Random variable	Distribution	Mean	Standard deviation
Wave height, H (m)	Normal	3	1
Period, T (s)	Normal	8	1
Storm surge, η (m)	Normal	1	0.5

$$L = T\sqrt{g(D_{wl} + \eta)} \tag{17}$$

Auxiliary Parameters

In addition to design variables and random variables, auxiliary parameters are considered. The values and meanings of the auxiliary parameters are gien in Table 3. The construction costs for stone and concrete per cubic meter were taken from Minguez et al. (2013). The currency conversion rate was USD 1 = RMB 6.7 on April 22, 2019. Based on the construction cost, the auxiliary coefficients are used to calculate the extra repair costs owing to different modes of failure. A sea state is, with respect to wind, waves, and swell, the general condition of the surface of the water. Generally, 100–1,000 wave periods are used for the duration of a sea state; 40 min was taken as the duration of a sea state in the present study. An experimental model was proposed by Castillo (2004) for R_u/H, where A_u and B_u are fitness coefficients of a rubble armor unit for run-up, and the values of the two parameters are given in Table 3. The design lifetime of a breakwater structure, in general, is between 25 and 50 years. Structure lifetime, mean period, water level, unit weight of different materials, and so on were considered (Table 3).

Cost Function

The initial construction cost function $f(x)$ is

$$C_{in} = \sum_{i=1}^{n} c_i v_i \tag{18}$$

where c_i = construction cost per unit volume for different structural parts; and v_i = volume for different structural parts.

The expected repair costs for overtopping failure, armor instability failure, crown wall sliding failure, and interactions among the three modes of failure are considered as

$$C_{re} = C_o + C_a + C_c + C_{oa} + C_{oc} + C_{ac} + C_{oac} \tag{19}$$

where

$$C_i = \lambda_i P_i D C_{in}$$
$$C_{ij} = \lambda_{ij} P_{ij} D C_{in}$$
$$C_{123} = \lambda_{123} P_{123} D C_{in} \tag{20}$$

where λ_i = partial coefficient that can affect the cost for the failure probability of only one failure mode; $P_i D$ = failure probability of only one failure mode during the whole lifetime; λ_{ij} = partial coefficient that can affect the cost for the failure probability of two different failure modes; $P_{ij} D$ = failure probability of two different failure modes during the whole lifetime; and subscripts $i = o, a, c$ represent overtopping failure, armor instability failure, and crown wall sliding failure, respectively.

Solution of Breakwater Example

According to the aforementioned formulas, the objective function of the optimal design problem for a rubble mound breakwater can be written

Table 3. Values used for auxiliary parameters

Parameter	Unit	Definition	Value
c_c	$\$/m^3$	Concrete construction cost per unit volume	60
c_s	$\$/m^3$	Stone construction cost per unit volume	2.4
D	Year	Structure lifetime	50
λ_o	—	Auxiliary coefficient used to calculate extra repair cost owing to Failure O	0.1
λ_a	—	Auxiliary coefficient used to calculate extra repair cost owing to Failure A	0.4
λ_c	—	Auxiliary coefficient used to calculate extra repair cost owing to Failure C	0.4
λ_{oc}	—	Auxiliary coefficient used to calculate extra repair cost owing to interaction between Failures O and C	0.15
λ_{ac}	—	Auxiliary coefficient used to calculate extra repair cost owing to interaction between Failures A and C	0.02
λ_{oa}	—	Auxiliary coefficient used to calculate extra repair cost owing to interaction between Failures O and A	0
λ_{oac}	—	Auxiliary coefficient used to calculate extra repair cost owing to interaction among Failures O, A, and C	0
d_{st}	min	Duration of sea states	40
$\bar{T}$	s	Mean period	8
A_u	—	Coefficient depending on armor units to calculate wave run-up	1.37
B_u	—	Coefficient depending on armor units to calculate wave run-up	–0.6
D_{wl}	m	Water level	10
g	m/s^2	Acceleration of gravity	9.81
γ_w	kN/m^3	Water unit weight	10.25
γ_s	kN/m^3	Rubble mound unit weight	26
γ_c	kN/m^3	Concrete unit weight	23.5
p	m	Upper crown wall width	1
s	m	Lateral crown wall height	1.5
μ_c	—	Friction coefficient	0.6

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$$\underset{F_c, \tan\alpha, d, A_c, n, e}{\text{Minimize}} \; C = (c_c v_c + c_s v_s)[1 + D(\lambda_o P_o + \lambda_a P_a + \lambda_c P_c + \lambda_{oa} P_{oa} + \lambda_{oc} P_{oc} + \lambda_{ac} P_{ac} + \lambda_{oac} P_{oac})] \quad (21)$$

subject to

$$v_c = 2d + 3(e - 1) \quad (22)$$

$$v_s = 487.5 + \frac{32}{\tan\alpha} \quad (23)$$

$$P_i = 1 - [1 - \Phi(-\beta_i)]^{d_{st}/\bar{T}} \quad (24)$$

$$P_{ij} = 1 - [1 - \Phi(-\beta_{ij})]^{d_{st}/\bar{T}} \quad (25)$$

$$P_{ijk} = 1 - [1 - \Phi(-\beta_{ijk})]^{d_{st}/\bar{T}} \quad (26)$$

where the objective function Eq. (21) is expressed considering failure probabilities; Eqs. (22) and (23) are obtained in terms of geometric relations; Eqs. (24)–(26) define the relationships between reliability indexes and failure probabilities (single waves and the sea state); $\Phi(\cdot)$ = standard normal cumulative distribution function; P_i = probability of failure; and β_i = reliability index.

From Eqs. (24)–(26), the probability of failure P_i is related to the reliability index β_i. For $i = o, a, c$, β_i is the reliability index related to the overtopping failure, armor instability failure, and crown wall sliding failure, respectively. For $i, j = o, a, c$, β_{ij} is the reliability index related to two of three modes of failure. For $i = o$, $j = a$, $k = c$, β_{ijk} is the reliability index related to all three modes of failure. The reliability index β_i with corresponding parameters for the modes of failure is described in the Appendix.

The formulas and parameters mentioned previously present a multidimensional constrained nonlinear optimization problem, which is solved using mathematical software MATLAB version R2016a. Every design variable is optimized from the initial value. Using the aforementioned formulas in the optimal design solution, the solver fmincon is used to solve the optimization problem, which considers functions including the objective function, constraint functions, and linear functions; values involving initial values, the lower boundary, and the upper boundary; and options consisting of the algorithm, scale, tolerance, and so on. Then the random variables follow a normal distribution.

Results and Discussion

To reduce the total cost of a rubble mound breakwater and ensure good performance, three failure modes and the interactions among them are considered. The optimal design results were compared with a practical engineering analysis using a numerical study.

Table 4 compares the failure probabilities among different modes of failure. All probabilities are yearly failure rates. From the comparison of every single mode of failure, the values of this paper are less than the corresponding values from Castillo et al. (2004), and the values from both the present study and Castillo et al. (2004) are less than the upper bound of failure (Castillo et al. 2006). Obviously, the optimal design satisfies limit-state equations for every single mode of failure. The probabilities of failure for the interactions of different failure modes indicate that one storm may induce more than one failure when sea waves in one storm occur. Moreover, different failure modes are not statistically independent, and due to common inducing agents, the breakwater is likely to suffer a progressive collapse from different failure modes. Therefore, it is essential to consider interactions between failure modes,

Table 4. Comparison of failure probabilities among modes of failure

Mode of failure	Probability of failure: Present study	Castillo et al. (2004)	Upper bound of failure (Castillo et al. 2006)
Failure O	0.002	0.01	0.005
Failure A	1.19×10^{-7}	0.001	0.003
Failure C	2.80×10^{-7}	0.001	0.001
Failures O + C	7.20×10^{-8}	—	—
Failures O + A	0	—	—
Failures C + A	1.18×10^{-9}	—	—
Failures O + A + C	0	—	—

Table 5. Illustration of iterative procedure

Variable	Iteration: 1	4	5	9 (end)
Cost ($)	2,411.7	2,027.6	2,184.9	1,951.4
Error	54.64	7.79	20.11	5.684×10^{-14}
F_c (m)	3.00	5.28	5.89	5.13
$\tan\alpha$	0.30	0.39	0.57	0.50
d (m)	5.00	3.90	3.93	4.00
A_c (m)	1.00	2.10	2.07	2.00
n (m)	3.00	2.93	5.07	5.00
e (m)	4.00	4.10	5.07	3.56

Table 6. Cost sensitivities $\partial C/\partial x$ in regard to variables

Variable	Δx	ΔC($)
D_{wl} (m)	1	62.82
α (degrees)	1	−115.66
d (m)	1	116.05
A_c (m)	1	−113.59
n (m)	1	−36.25
e (m)	1	59.21

and the solution of optimal design for a rubble mound breakwater in this study is more economical and more reliable.

Table 5 gives the iterative procedure of the final cost and the design variables. The optimization process described the evolution of the convergence process in nine iterations. The admissible tolerance of the convergence method was $\varepsilon = 1 \times 10^{-6}$. The total expected cost using the optimal design solution was $C = \$1951.44$, which was 19.1% less than the design cost before optimization. Every variable value in the optimal method is listed in Table 5.

The cost sensitivity $\partial C/\partial x$ represents the relationship between the total cost and the variables in the breakwater example (Table 6). Table 6 lists the corresponding increase in total cost when one variable increases by one unit; ΔC indicates a mean value of cost sensitivities due to the complexity of the interactions between different parameters. As an example, if the water level increases by 1 m, and all other variables remain the same, the cost will increase by $62.82 based on the original value. The increased cost is based on the optimal design and the sufficient performance of the breakwater with no failure. For variables α, A_c, and n, the negative value of total cost represents the decreasing cost with the variable increasing by one unit. The cost increments for variables α, A_c, and d are larger than those for other variable (Table 6). Thus, to reduce the cost as much as possible, variables α and A_c should be increased

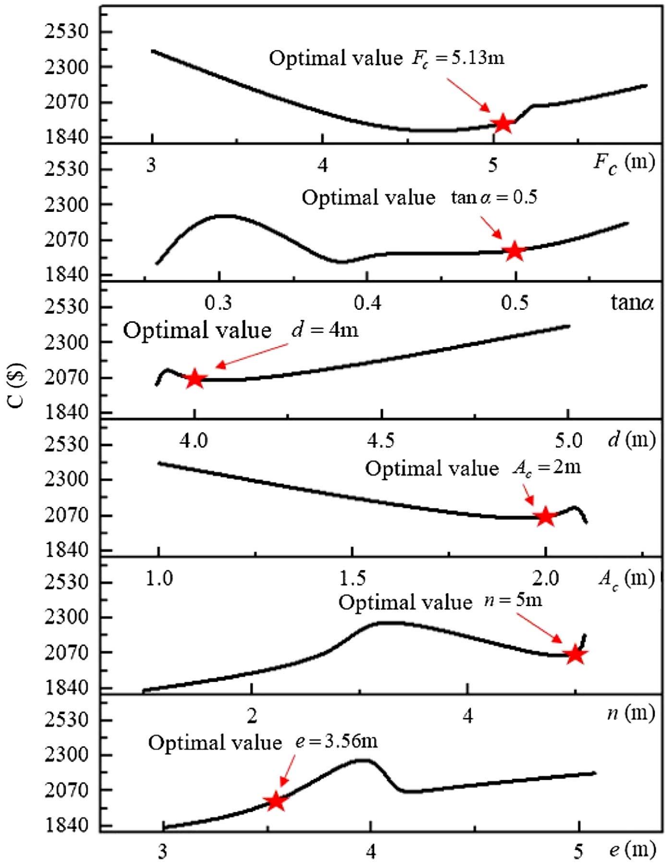

Fig. 6. Multicurve stack plot of the total cost for design variables. Stars correspond to optimal points of different design variables.

and variable d should be decreased without reducing the performance of a breakwater.

Fig. 6 shows the total cost stack plot with regard to the different design variables, where the star denotes the optimal point in the optimization process. The total cost monotonously increases as the slope angle increases. The total cost monotonously decreases with increasing height of the crown wall. In addition, the total cost fluctuates according to other variables. Obviously, the optimal values of the design variables clearly correspond to the optimal value of total cost from the stars, but the optimal value of total cost is not the minimum of total cost from Fig. 6. This is because guaranteeing the stability and reliability of the structure is the necessary prerequisite for obtaining optimal value.

Conclusions

An optimal probabilistic method was used to describe the total cost for a structure under wave action, in which three modes of failure and their interactions are involved. In this case, three modes of failure were introduced for rubble mound breakwaters: Failure O (overtopping failure), Failure A (armor instability failure), and Failure C (crown wall sliding failure). In addition, a new parameter illustrates the extent to which the failure mode (or interaction) affects the structure, which can represent the response of interaction in the optimal design cost.

Based on the optimal design method and the interactions between failure modes, a complete and comprehensive framework was developed for the optimized total cost of a rubble mound breakwater. In the proposed framework, the optimal design satisfies the limit-state equation for every single mode of failure, and the interactions among the modes of failure included in the optimization process for rubble mound breakwaters provide a route for considering the total cost clearly. Moreover, the optimization solution was analyzed, interpreted, and verified by means of a practical engineering example of a rubble mound breakwater. The probabilities for the different modes of failure were calculated in this paper.

A practical rubble mound breakwater example indicated that the total expected cost using the optimal design solution was 19.1% less than the design cost before optimization. The sensitivity analysis demonstrated that variables d and e, i.e., the size of the crown wall, should be decreased to reduce the cost as much as possible without reducing the performance of the breakwater.

From the aforementioned example, the proposed framework can be applied to rubble mound breakwaters of different types. Other coastal structures suffer the same failure modes, and so also could be optimized by means of the framework. The solution of this study can provide economical design recommendations for different coastal structures guaranteeing the reliability of the structure.

Appendix. Reliability Index

Because they are easy to carry out and are customarily used in the reliability analysis method, the formulas of Minguez et al. (2013) were used here. The reliability index β_1 for the modes of failure is expressed as

$$\beta_i = \min_{H,T,\eta} \sqrt{H^2 + T^2 + \eta^2} \tag{27}$$

For overtopping failure, β_o is subject to

$$\frac{R_u}{H} = A_u(1 - e^{B_u I_r}) \tag{28}$$

$$I_r = \tan\alpha / \sqrt{H/L} \tag{29}$$

$$L = T\sqrt{g(D_{wl} + \eta)} \tag{30}$$

$$F_c = R_u \tag{31}$$

where the reliability index is calculated using Eq. (27), and Eqs. (28)–(31) are the constraints related to overtopping failure; H, T, and η = random variables (Table 2); R_u = wave run-up; A_u and B_u = coefficients depending on the armor units to calculate the wave run-up; I_r = Iribarren's number; $\tan\alpha$ = slope angle; L = wave length; D_{wl} = water level; and F_c = freeboard.

For armor instability failure, β_a is subject to

$$W = \gamma_w R \Phi_e H^3 \tag{32}$$

$$\Phi_e = A_r(I_r - I_{r_0})e^{B_r(I_r - I_{r_0})} \tag{33}$$

$$R = \frac{\gamma_s}{\gamma_w\left(\frac{\gamma_s}{\gamma_w} - 1\right)^3} \tag{34}$$

$$A_r = 0.2566 - \frac{0.177}{\tan\alpha} + \frac{0.034}{(\tan\alpha)^2} \tag{35}$$

$$B_r = -0.0201 - \frac{0.4123}{\tan\alpha} + \frac{0.055}{(\tan\alpha)^2} \tag{36}$$

$$I_r \geq I_{r_0} \tag{37}$$

$$I_r = \tan\alpha/\sqrt{H/L} \quad (38)$$

$$I_{r_0} = 2.656 \tan\alpha \quad (39)$$

where Eqs. (32)–(39) are the constraints associated with armor instability failure; W = weight of armor pieces; γ_w = water unit weight; R = dimensionless constant; Φ_e = stability function; A_r and B_r depend on $\tan\alpha$ in terms of approximate experimental relations, i.e., Eqs. (35) and (36); I_{r_0} = Iribarren's number for shallow water conditions; and γ_s = rubble mound unit weight.

For crown wall sliding failure, β_c is subject to

$$\mu_c(W_1 - F_v) = F_h \quad (40)$$

$$W_1 = v_c\gamma_c - W_f e\gamma_w \quad (41)$$

$$F_v = \frac{1}{2}\lambda P_{S_0} e \quad (42)$$

$$F_h = (S_0 - A_c)P_{S_0} + (W_f + A_c)\lambda P_{S_0} \quad (43)$$

$$I_r \geq 2 \quad (44)$$

$$P_{S_0} = \alpha\gamma_w S_0 \quad (45)$$

$$S_0 = H\left(1 - \frac{A_c}{R_u}\right) \quad (46)$$

$$\alpha = 2C_f\left(\frac{R_u}{H}\cos\alpha\right)^2 \quad (47)$$

$$\lambda = 0.8\exp\left(-10.9\frac{b}{L}\right) \quad (48)$$

$$v_c = pd + s(e - p) \quad (49)$$

$$C_f = 1 + (\tan\alpha)^{1.2} \quad (50)$$

where Eqs. (40)–(50) are the constraints related to crown wall sliding failure; μ_c = friction coefficient; W_1 = actual crown weight; F_v = total vertical force because of water pressure; F_h = total horizontal force because of water pressure; γ_c = concrete unit weight; W_f = submerged height of crown wall; e = crown wall width; P_{S_0} = wave water pressure; S_0 = wave height due to run-up; A_c = berm level; α = random nondimensional variable; C_f = experimental random coefficient; and d = sea-side berm width.

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